Joint Probability

- example: insurance policy deductibles

y (home)

x (automobile)	**$0**	**$100**	**$200**
$100	0.20	0.10	0.20
$250	0.05	0.15	0.30

Joint Probability and Independence

- X and Y are said to be independent if

$$P(x,y) = P(x)\ P(y)$$

for all possible values of x and y

- example: two fair dice

$$P(X\text{=even and }Y\text{=even}) = (1/2)\ (1/2)$$
$$P(X\text{=1 and }Y\text{=not 1}) = (1/6)\ (5/6)$$

- are X and Y independent in the insurance deductible example?

Marginal Probabilities

- the marginal probability distribution of X

$$P_X(x) = \sum_y P(x, y)$$

describes the probability of the event that X has the value x

- similarly, the marginal probability distribution of Y

$$P_Y(y) = \sum_x P(x, y)$$

describes the probability of the event that Y has the value y

Joint Probability

- example: insurance policy deductibles

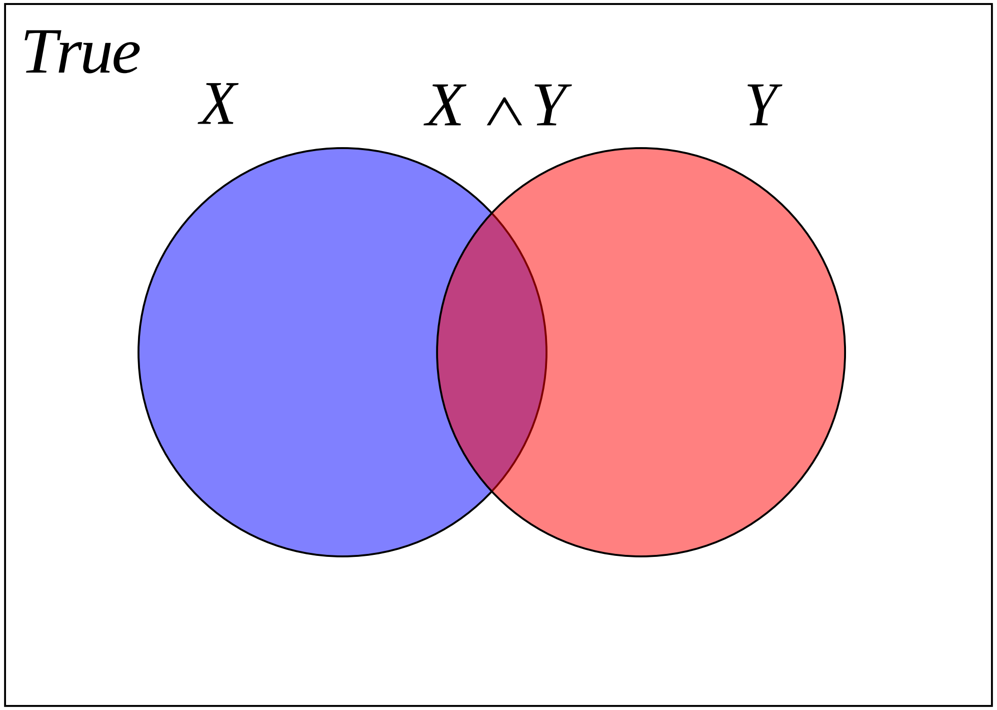
y

x		$0	$100	$200
		$0	**$100**	**$200** ← home
	$100	0.20	0.10	0.20
	$250	0.05	0.15	0.30

↑ automobile

Conditional Probability

- the conditional probability $P(x \mid y) = P(X{=}x \mid Y{=}y)$ is the probability of $P(X{=}x)$ if $Y{=}y$ is known to be true
  - "conditional probability of x given y"



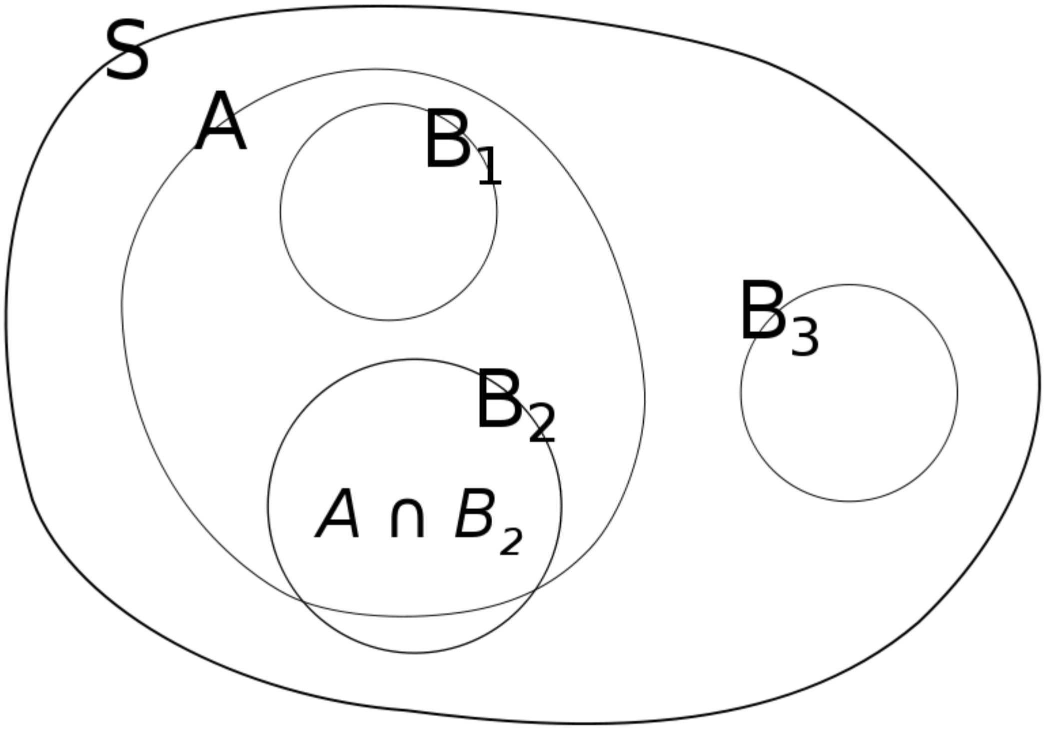
Conditional Probability

$$P(A) \approx 0.3$$

$$P(A \mid B_3) = ?$$

$$P(A \mid B_1) = ?$$

$$P(A \mid B_2) = ?$$



Conditional Probability

- "information changes probabilities"
- example:
  - roll a fair die; what is the probability that the number is a 3?
  - what is the probability that the number is a 3 if someone tells you that the number is odd? is even?
- example:
  - pick a playing card from a standard deck; what is the probability that it is the ace of hearts?
  - what is the probability that it is the ace of hearts if someone tells you that it is an ace? that is a heart? that it is a king?

Conditional Probability

$$P(x \mid y) = \frac{P(x, y)}{P(y)}$$

- if X and Y are independent then

$$P(x, y) = P(x)P(y)$$

$$\therefore P(x \mid y) = \frac{P(x)P(y)}{P(y)} = P(x)$$

Bayes Formula

$$P(x,y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$\boxed{\underbrace{P(x \mid y)}_{\text{posterior}} = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$$

Bayes Rule with Background Knowledge

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

Back to Kinematics

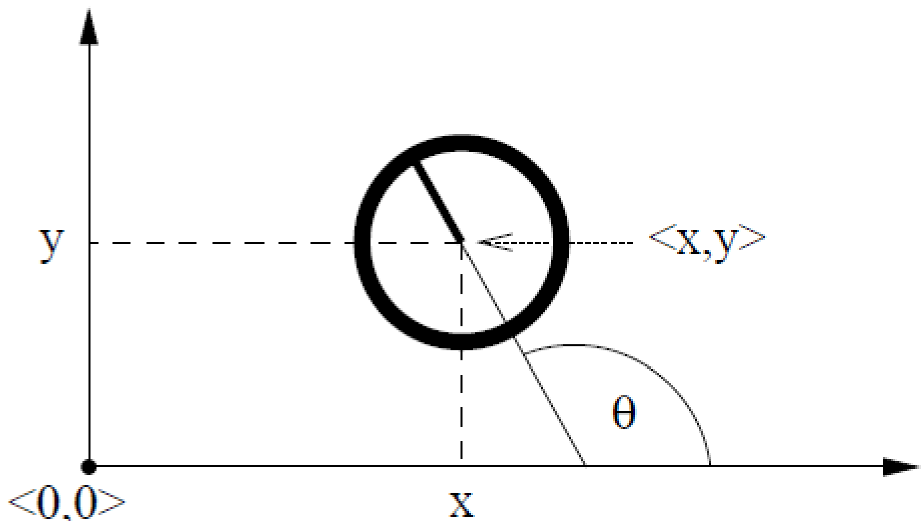


Figure 5.1 Robot pose, shown in a global coordinate system.

pose vector or state $x_t = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$

$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ location (in world frame)

$\left.\theta\right\}$ bearing or heading

Probabilistic Robotics

- we seek the conditional density

$$p(x_t \mid u_t, x_{t-1})$$

- what is the density of the state

$$x_t$$

  given the motion command

$$u_t$$

  performed at

$$x_{t-1}$$

Probabilistic Robotics

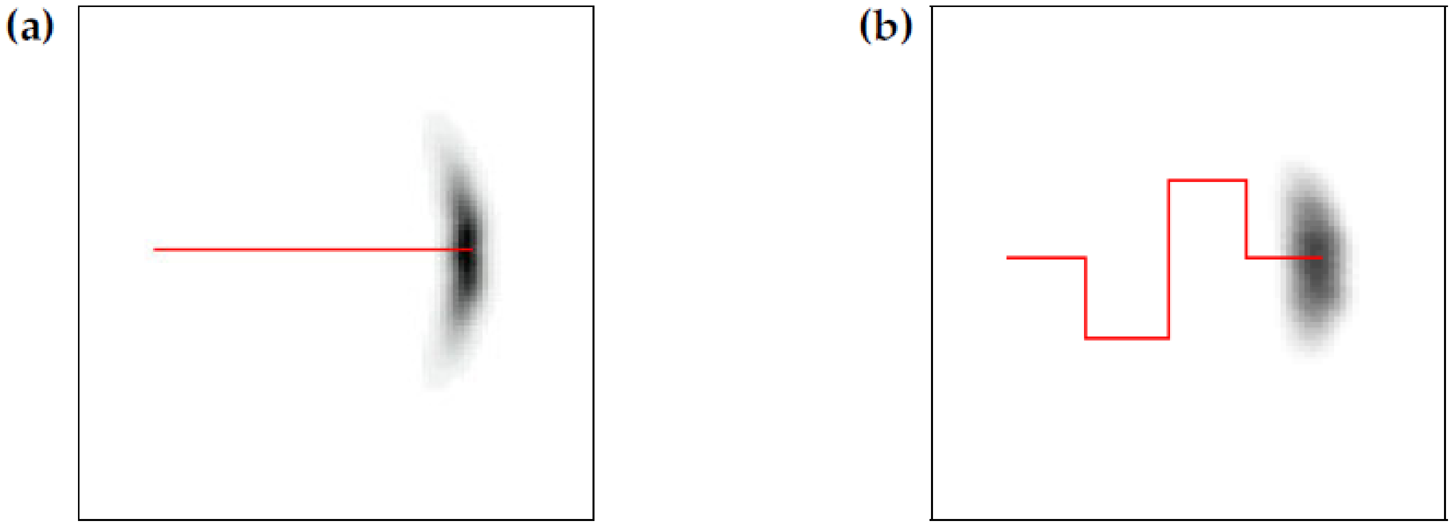


Figure 5.2 The motion model: Posterior distributions of the robot's pose upon executing the motion command illustrated by the solid line. The darker a location, the more likely it is. This plot has been projected into 2-D. The original density is three-dimensional, taking the robot's heading direction θ into account.

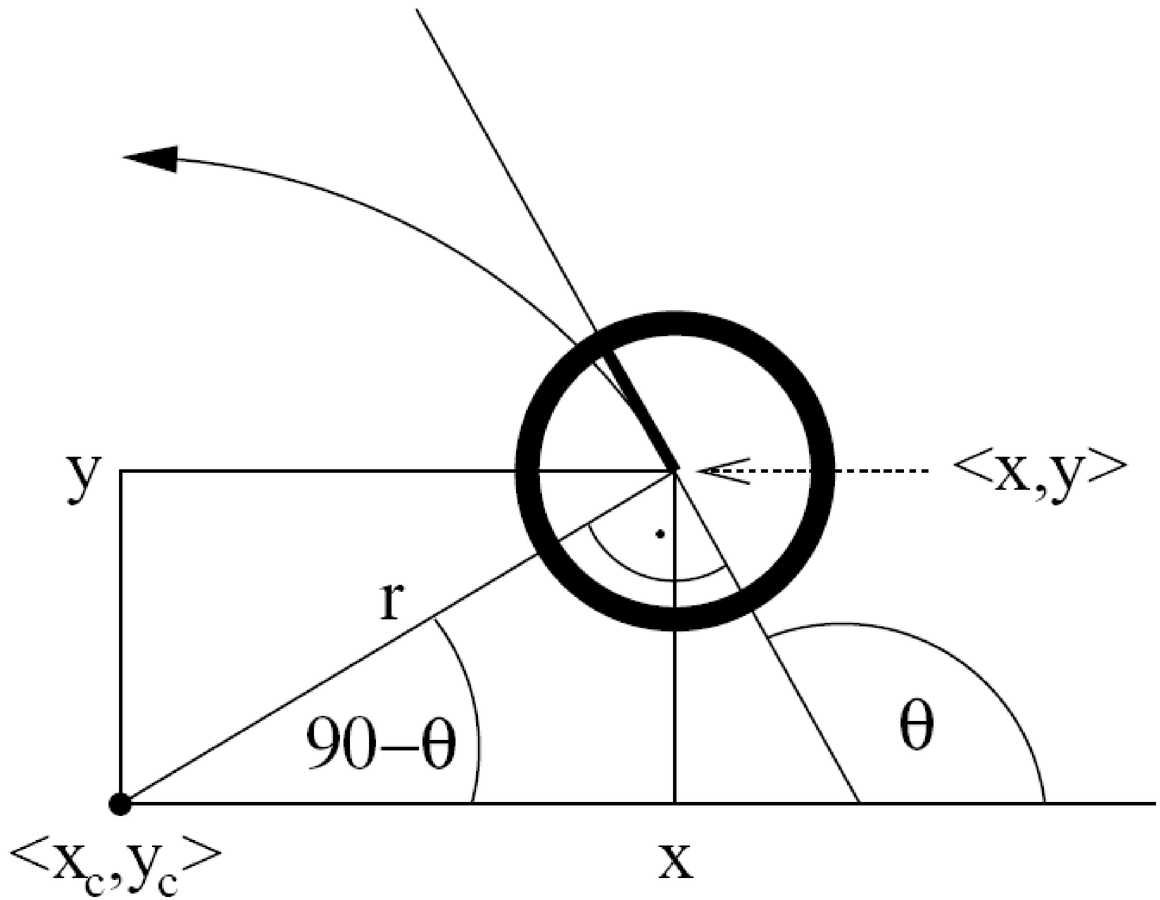
Velocity Motion Model

- assumes the robot can be controlled through two velocities
  - translational velocity v
  - rotational velocity ω
- our motion command, or control vector, is

$$u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix}$$

- positive values correspond to forward translation and counterclockwise rotation

Velocity Motion Model



Velocity Motion Model

- center of circle

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\lambda \sin\theta \\ \lambda \cos\theta \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y-y') \\ \frac{y+y'}{2} + \mu(x'-x) \end{pmatrix}$$

where

$$\mu = \frac{1}{2} \frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

Velocity Motion Model

Algorithm motion_model_velocity(x_t, u_t, x_{t-1})**:**

$$\mu = \frac{1}{2}\ \frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta - (x - x')\sin\theta}$$

$$x^* = \frac{x + x'}{2} + \mu(y - y')$$

$$y^* = \frac{y + y'}{2} + \mu(x' - x)$$

$$r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2}$$

$$\Delta\theta = \mathrm{atan2}(y' - y^*, x' - x^*) - \mathrm{atan2}(y - y^*, x - x^*)$$

$$\hat{v} = \frac{\Delta\theta}{\Delta t}\ r^*$$

$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\hat{\gamma} = \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$$

return $\mathbf{prob}(v - \hat{v}, \alpha_1|v| + \alpha_2|\omega|)\ \cdot\ \mathbf{prob}(\omega - \hat{\omega}, \alpha_3|v| + \alpha_4|\omega|)$
$\cdot\ \mathbf{prob}(\hat{\gamma}, \alpha_5|v| + \alpha_6|\omega|)$